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# ADAPTIVE CONTROL OF A BINARY DISTILLATION COLUMN USING QUADRATIC PROGRAMMING

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**Abstract**— A quadratic cost function for the optimal control of binary distillation column was minimized using quadratic programming. An input-output model was used and its parameters were recursively updated by instrumental variable method.

A control experiment was conducted for the performance appreciation of the control technique. The performance of the adaptive control was found to be much improved comparing with the conventional PID control. The technique was capable to obtain fast solution and easy to apply for the multi-input-multi-output systems. And also the algorithm was stable for a long term operation which is very critical for the industrial application.

## INTRODUCTION

Ever since the computer has been introduced in the chemical process industry, its use is almost unlimitedly expanded. In the application to the process control, it has induced a great deal of improvement of the control technology. While a conventional feedback control uses the process error from the set point to calculate the manipulated input, an advanced control utilizes a mathematical model of the process to find the input predictively. It requires computer in order to solve the model and the control equation. As the fast and inexpensive computers have been developed, more complex models can be applied in the control calculation. However, fast changing systems, e.g., reactors and furnaces, allow only very short period of time to get new values of the manipulated input. For this reason, the rigorous process models are not recommended for the application in the chemical process control.

Many studies on the control of distillation column have been conducted with the adaptive model, and various types of objective function to minimize the deviation of process output have been introduced. The techniques of the self-tuning controller and self-tuning regulator based on quadratic cost function had been applied in the control of distillation column. Chien et al. [1] employed a self tuning control technique with time delay compensation in the dual composition control of a pilot scale distillation column, and found that it reduces the interaction between loops significantly. Another study [2] with self-tuning control strategy also reported that the control performance is much improved compared with the conventional control. A simple self-tuning regulator [3] was used in the control of top product composition of a binary distillation, and proved its adaptability to the inherent nonlinear behavior of the system as well as a good control performance. The same technique has been utilized in the distillation column control by Dahlqvist [4], and he obtained the similar results.

The control technique of the pole placement design using stable feedback gain was tested in the binary distillation column control [5], and the study showed that the adaptive controller is easy to implement and gives a noticeable improvement over the conventional PID controller. Another adaptive method, the model reference adaptive control, was applied in the control of binary distillation column by Martin-Sanchez and Shah [6]. From the study, not only its adaptability and handling capability of the system interactions but also the excellent performance in the regulatory control has been indicated.

The quadratic programming to solve the quadratic cost function has been applied to the dynamic matrix control [7,8] and the internal model control [9]. Since the problem can be transformed into the linear programming problem, its solution can be obtained fast. It is a big advantage in the process control, and it has been proved in the previous studies.

In this study an adaptive input-output model was

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utilized to the quality control of both top and bottom product of binary distillation column. The model parameters were updated by instrumental variable method. The quadratic cost function was minimized using quadratic programming, and at the same time process limitations were included in the calculation of the manipulated variable. An experimental evaluation of the control performance was conducted and the performance was compared with the conventional feedback control.

## CONTROL ALGORITHM

The quadratic cost function has been very effectively used as an objective function in the most of control techniques. However, a problem comes from the fact that the quadratic objective function is nonlinear and requires nonlinear optimizing technique to minimize the objective function, even though the original model of the process is linear. The quadratic programming problem can be solved by transforming it into a linear programming problem applicable by the simplex algorithm.

The input-output presentation of the process in a discrete form is

$$A(q^{-1})y(k)=B(q^{-1})q^{-d}u(k) \quad (1)$$

where

$$A(q^{-1})=1+a_1q^{-1}+\cdots+a_nq^{-n}$$
$$B(q^{-1})=b_1q^{-1}+\cdots+b_mq^{-m}$$

The coefficients, $a_1 \cdots a_n$, $b_1 \cdots b_m$, can be renewed at every time step by various recursive identification methods.

The instrumental variable method [10] has been successfully applied in many adaptive control studies. The instrumental variable, z(k), is recursively calculated from

$$z(k)=\psi(k-1)\mathbf{Z}(k-1)$$

where $\mathbf{Z}(k\text{-}1) = [z(k\text{-}1),\cdots,z(k\text{-}n), u(k\text{-}d\text{-}1),\cdots,u(k\text{-}d\text{-}m)]^T$

The parameter vector $\psi(k)$ can be obtained from

$$\psi(k)=\psi(k-1)+\frac{\mathbf{P}(k-1)\mathbf{Z}(k-1)}{1+\mathbf{Y}^T(k-1)\mathbf{P}(k-1)\mathbf{Z}(k-1)} \times [y(k)-z(k)]$$

and

$$\mathbf{P}(k)=\mathbf{P}(k-1) - \frac{1}{\lambda}\frac{\mathbf{P}(k-1)\mathbf{Z}(k-1)\mathbf{Y}^T(k-1)\mathbf{P}(k-1)}{1+\frac{1}{\lambda}\mathbf{Y}^T(k-1)\mathbf{P}(k-1)\mathbf{Z}(k-1)} \quad (3)$$

where $\mathbf{Y}(k\text{-}1) = [y(k\text{-}1),\cdots,y(k\text{-}n), u(k\text{-}d\text{-}1),\cdots,u(k\text{-}d\text{-}m)]^T$.

The quadratic cost function for the multi-input-multi-output system is

$$J=E\{[\mathbf{y}(k)-\mathbf{y}_s(k)]^T\mathbf{Q}[\mathbf{y}(k)-\mathbf{y}_s(k)] + \mathbf{u}^T(k-d-1)\mathbf{R}\mathbf{u}(k-d-1)\} \quad (4)$$

When the delay matrix, composed of delays between inputs and outputs, is not singular, a set of inputs to minimize Eq. (4) can be obtained [6]. Substituting Eq. (1) into Eq. (4) gives a simple quadratic objective function, since the process values of input and output of past steps are known. For the simplicity step indication can be dropped.

$$J=E\{[\mathbf{bu}+y_c]^T\mathbf{Q}[\mathbf{bu}+y_c]+\mathbf{u}^T\mathbf{Ru}\} \quad (5)$$

A minimizing problem of the objective function, Eq. (5), with process constraints

$$\mathbf{fu}\leq\mathbf{e}, \quad (6)$$

can be solved using quadratic programming [11]. The minimizing objective equation is transformed into maximizing objective as follows:

$$G=\mathbf{c}^T\mathbf{u}-\frac{1}{2}\mathbf{u}^T\mathbf{qu} \quad (7)$$

subject to

$$\mathbf{fu}\leq\mathbf{e}$$

Apply the Kuhn-Tucker condition to the minimizing problem of Eq. (7) results in a linear programming problem.

Minimize

$$g=\sum_j w_j \quad (8)$$

subject to

$$\mathbf{qu}+\mathbf{f}^T\mathbf{v}-\mathbf{v}+\mathbf{w}=\mathbf{c}$$
$$\mathbf{f}^T\mathbf{u}+\mathbf{u}=\mathbf{e}$$
$$\mathbf{u}^T\mathbf{v}\geq 0$$
$$\mathbf{w}\geq 0$$
$$\mathbf{u}^T\mathbf{v}=0.$$

The linear problem of Eq. (8) can be readily solved with simplex algorithm.

The solution is assured as long as the objective function has a minimum value for the bounded input variable. The convexity of the objective function is guaranteed when the **q** coefficient matrix of Eq.(7) is positive definite. This gives the robustness of the controller. Since the weight matrices in Eq. (4) are composed of positive real numbers and the model parameters are valid, the condition is always satisfied.

Since the algorithm calculates the control input for only one step and minimizes the objective function at one step, the input changes too much and it is undesirable in the real process. It was taken care of by implementing only a portion of input change. In this study 30 percent of variation was applied by considering the delay of bottom temperature response from the variation of steam flow rate. From the preliminary experiment it has been observed that the steam flow rate has stronger influence on both outputs than the other input and the 30 percent application gives the best performance.

## EXPERIMENTAL

A distillation column of six inches in diameter and 10 trays, which has 4 bubble caps in each tray, is used in the control experiment. The detailed schematic diagram is shown in Figure 1. For the control experiment, six control loops are used as indicated in the figure. Among them two temperature control loops for PID control has been turned into the flow control loops when the adaptive control technique is applied. The flow rates of top and bottom products are decided from the level of reflux drum and reboiler respectively. The column pressure is adjusted by controlling the flow rate of cooling water in the condenser. The feed flow rate is maintained with a single loop feedback control.

The temperature was measured with a platinum resistance thermometer and its resistance was converted into voltage signal for the data acquisition. The flow rate was determined by the differential pressure from an orifice flowmeter, and a d/p transducer was installed to obtain the voltage signal. A pressure transducer was used for the measurement of the column pressure in the same manner. The liquid level of reflux drum and reboiler was measured with a float having a piece of iron wire of which the location was detected with a transducer made of a small transformer through a glass tube.

A motor driven ball valve was employed for the control of liquid flow. The motor can rotate in two directions and two small push buttons were attached for the limit of clockwise and counter-clockwise turns. The turning angle is decided by the on-signal time from the controller.

An IBM PC/XT compatible microcomputer and a 32 bit microcomputer were connected to the experimental system. They communicate asynchronously each other. The former is called a slave controller for the basic control which takes care of collecting the process values and manipulating the control valves, and the latter for the parameter estimation and control calculation. Basic control includes PID control of six inde-

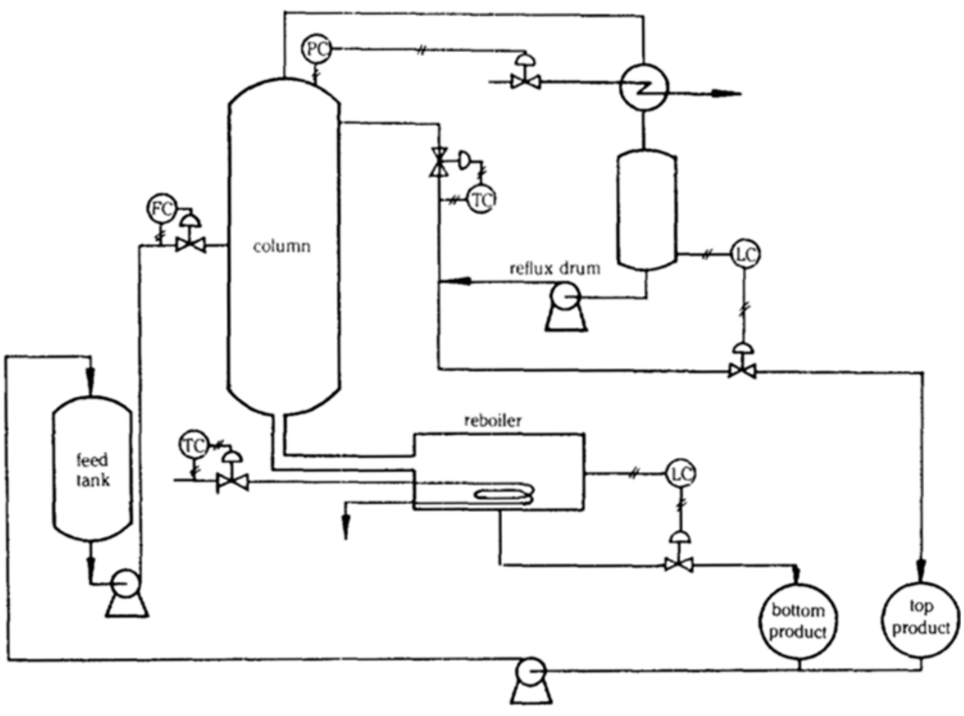


**Fig. 1. Schematic diagram of experimental setup.**

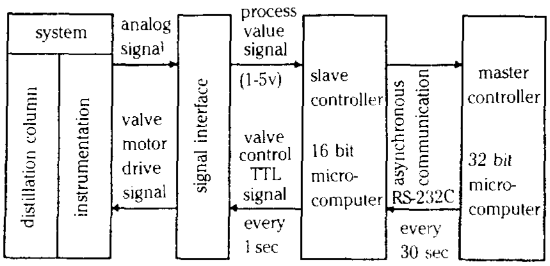


**Fig. 2. Block diagram of control system structure.**

pendent feedback loops, the transfer of process data to the other computer, master controller, and the reception of new set point of manipulated variable from the master controller while the adaptive control scheme is applied. In PID control, the set points of top and bottom temperatures are given by operator and the master computer only accepts the process values. The sequence of signal processing is given in Figure 2.

A mixture of methanol and water was provided for a feed, and its methanol concentration was 52 wt%. The concentration of top and bottom product was calculated from the temperature using Wilson equation and compared with the measured value by the refractive index. The specification of product from the distillation is usually given as a concentration, but the analysis takes to much time and it limits the step time of the control cycle. For the control purpose, the temperatures of top and bottom trays can replace the concentration specifications instead, when the column pressure remains constant. Since the reflux drum was open to atmosphere and the pressure was controlled by the flow rate of cooling water, the column pressure was nearly unchanged throughout the experiment.

## RESULTS AND DISCUSSION

The model of the binary distillation system consists of two manipulated variables, reflux and steam flow rates, and two controlled variables, top and bottom temperatures of the column. Feed flow rate is also considered as disturbance input.

The order of the model was adopted as $n = 2$ and $m = 2$ in Eq. (1). The time delay steps were determined from a preliminary experiment and listed in Table 1. The sampling time was selected to be 30 seconds considering the model size and the response of the system. The calculation at each step took 4.3 seconds for the parameter estimation and 0.6 seconds for the control. The initial parameters for the recursive parameter estimation were obtained from the test operation and the values are listed in Table 2. The covariance matrix had the initial value of 0.00001 in its diagonal positions and rest of elements have zero value. Compared with other studies, the diagonal values are quite small. It is because the magnitude of process value in this study is higher than those of others. The forgetting factor was set at 0.99, and these values are also shown in Table 2.

**Table 1. Model parameters**

| parameter | value |
|---|---|
| n | 2 |
| m | 2 |
| d for $y_1$ | $0(u_1), 2(u_2), 9(u_3)$ |
| d for $y_2$ | $8(u_1), 2(u_2), 4(u_3)$ |

**Table 2. Initial model coefficients, estimation parameter and weight matrices**

| | |
|---|---|
| **a** | $\begin{bmatrix} 0.18 & -1.14 & -0.067 & 0.04 \\ -0.011 & -0.012 & -0.27 & -0.72 \end{bmatrix}$ |
| **b** | $\begin{bmatrix} -0.014 & -0.019 & 0.016 & 0.019 & -0.019 & 0.017 \\ -0.003 & 0.001 & -0.005 & 0.008 & -0.008 & -0.006 \end{bmatrix}$ |
| $\lambda$ | 0.99 |
| **P** | 0.00001 **I** |
| **Q** | $\begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$ |
| **R** | $\begin{bmatrix} 1/900 & 0 \\ 0 & 1/1600 \end{bmatrix}$ |

The weight matrices in the control objective function were determined to adjust the magnitude of each term and their values are shown in Table 2.

The constraints of the system were composed of the limitations of the process which are the maximum available steam flow rate and reflux flow rate.

The experiments were conducted for both the conventional PID control and the adaptive control using quadratic programming, and the performances of both cases were compared. The tuning parameters were obtained from the open loop response and adjusted for the nonlinearity of the control valve by a trial-and-error method. The settings were: $K_c = -2.5$, $\tau_I = 8$ min, $\tau_D = 0.1$ sec for the distillate loop; $K_c = 12$, $\tau_I = 1.5$ min, $\tau_D = 0.2$ sec for the bottom loop. The performance of PID control for the change of the set point is given in Figure 3. The top figure shows the variation of top tray temperature along with the sequence of time step. The next one is for the bottom temperature. The solid lines show the variation of top and bottom temperatures and the dotted lines are the set points. The set points were changed 2 degrees up and down for the top tray temperature and down and up for the bottom temperature. The third and bottom figures are for the

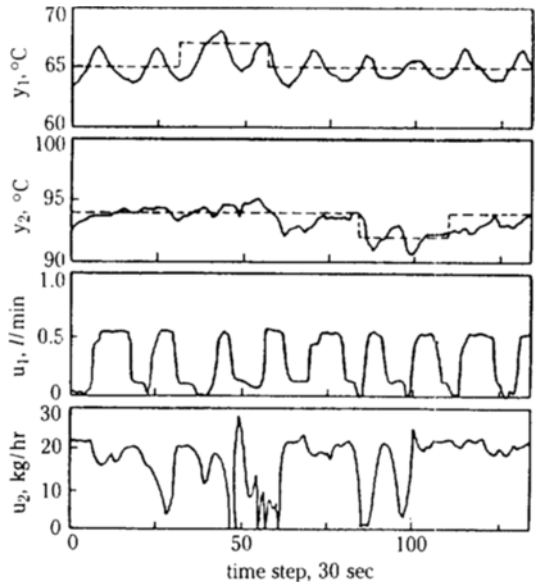


**Fig. 3. Response of top and bottom temperatures in PID control with step change of set point.**

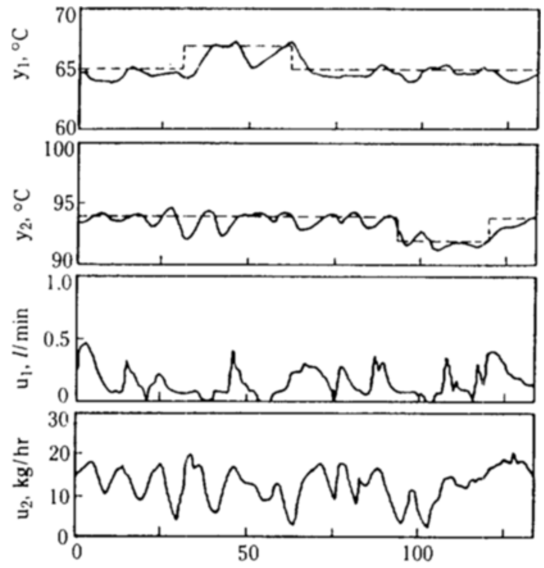


**Fig. 4. Response of top and bottom temperatures in adaptive control with step change of set point.**

reflux flow rate and steam flow rate. In this case, top tray temperature is controlled by reflux flow rate and bottom temperature is by the steam flow rate. However, two independent control loops do not utilize the information of each other, even though the two loops are systematically coupled. The effect of coupling in both loops can be seen most significantly when the set point of top temperature is varied. The bottom temperature deviates from its constant set point, when the top tray set point is changed. The steam flow is only determined by the bottom temperature and the steam is not raised unless the bottom temperature is below the set point, although the top temperature is way below the set point. It indicates that the process is coupled.

As can be seen in the figure, the top temperature is very poorly controlled, while the bottom temperature is adjusted in better shape. The holdup of top tray is much less than that of reboiler, and its small holdup is responsible for the high deviation in top tray temperature. In other word the state of top tray is prone to changes in disturbances.

In the similar situation, the result of the adaptive control is given in Figure 4. The steam flow is decided by both errors in top and bottom temperatures, and the two temperatures are controlled much better than in the case with PID control. The only problem can be observed in the top temperature variation when the set point of top tray temperature is raised from 65°C to 67°C. The temperature is off from the set point since

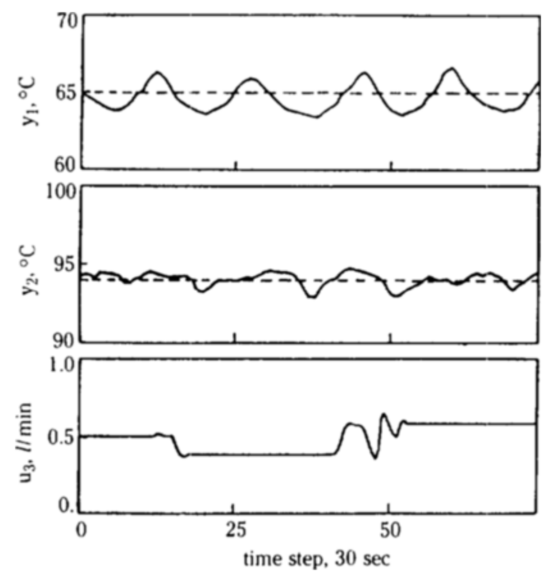


**Fig. 5. Response of top and bottom temperatures in PID control with variation of feed flow rate.**

the steam is reduced to adjust bottom temperature. However, the duration with high deviation is shorter than that in PID control.

In the adaptive control the set points are given to the master controller and the result of control calculation is fed to the slave controller as the set point of manipulated variable.

The regulatory performance of PID control is

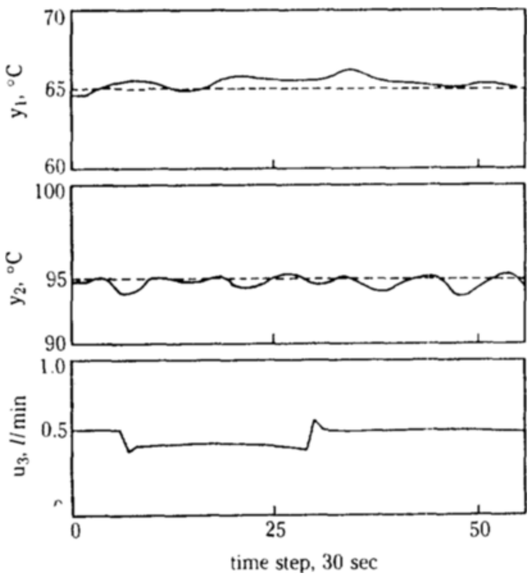


**Fig. 6. Response of top and bottom temperatures in adaptive control with variation of feed flow rate.**

shown in Figure 5. While the feed flow rate varies, the top and bottom temperatures are presented along with the set points, represented as dotted lines. Both top and bottom temperatures deviate very much from the set points. On the other hand, the performance of the adaptive control is significantly better than that of PID control for both top and bottom temperatures, as shown in Figure 6. The variation of feed flow rate in both cases has the same pattern, but the outcome of the adaptive control is better than that of PID control. It says that the adaptive control works very well in the regulatory control as well as in the servo control.

For the numerical comparison of the performances of PID control and adaptive control, the integral of absolute error (IAE) is summarized in Table 3. The adaptive control showed very good performance especially in the regulatory control.

The optimization problem having constraints is more difficult to solve than that without constraints. However, the problem in the chemical processes includes the constraints, and most of previous adaptive techniques do not manage the constraints. In that sense, it is a great advantage that the quadratic programming can handle the constrained problems.

In Ricker's study [9], three solution techniques were mentioned for the initial feasible solution. One of the methods, which is that the initial set of feasible solution is from the slack variables, was used in this study, and it was fast enough to calculate the control input.

The initial set of model parameters is very important in the adaptive model based applications. The stability in long term application solely depends on it. Especially highly noisy signals with the nonlinearity and uncertainty in the model result in the divergence of the parameters. In this study a set of updated parameters from the preliminary operation of the experimental column was introduced at the beginning stage, and the parameters were varied in the bounded range throughout the experiment. It proves that the technique can be utilized in the industrial applications.

**Table 3. Integral of absolute errors (IAE)**

| case | | PID | | adaptive | |
|---|---|---|---|---|---|
| | | no. of time steps | IAE | no. of time steps | IAE |
| set point change | | | | | |
| top set point | $y_1$ | 70 | 78.8 | 77 | 62.9 |
| | $y_2$ | 70 | 37.2 | 77 | 36.8 |
| bottom set point | $y_1$ | 65 | 43.9 | 58 | 21.4 |
| | $y_2$ | 65 | 50.0 | 58 | 31.8 |
| feed flow rate | $y_1$ | 75 | 61.8 | 57 | 26.1 |
| change | $y_2$ | 75 | 27.1 | 57 | 18.5 |

## CONCLUSION

A technique of the adaptive optimal control using quadratic programming was composed, and its performance was tested for the control of a binary distillation column. The result was compared with the conventional PID control.

The scheme showed a good performance compared with the PID control, and found to be suitable for the multi-input-multi-output system. The algorithm was stable for a long term operation. The technique is very fast in the calculation of the manipulated variable, and it can be applied in the fast systems and nonlinear systems. Since the process model and solution technique are simple and easy, the industrial application of the technique will be available.

## ACKNOWLEDGEMENT

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## NOMENCLATURE

A : output polynomial

a : coefficient in output polynomial

B : input polynomial
b : coefficient in input polynomial
**c** : coefficient matrix in Eq. (7)
d : delay step
**e** : constant matrix in constraint of Eq. (7)
**f** : coefficient matrix in constraint of Eq. (7)
G : objective function in Eq. (7)
g : modified objective function in Eq. (8)
J : cost function
$K_c$ : proportional gain
k : time step
m : input model order
n : output model order
**P** : covariance matrix
**Q** : weight matrix in cost function
**q** : coefficient matrix in Eq. (7)
**R** : weight matrix of loss term in cost function
u : input
$u_1$ : reflux flow rate, $l$/min
$u_2$ : steam flow rate, kg/hr
$u_3$ : feed flow rate, $l$/min
**v** : Lagrange multiplier
w : artificial variable
**y** : output
$y_1$ : top tray temperature, °C
$y_2$ : bottom temperature, °C

### Subscripts

c : constant
s : set point

### Greek Letters

$\lambda$ : forgetting factor
$\tau_D$ : derivative time
$\tau_I$ : reset time
$\psi$ : parameter vector

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